

**ON THE WELL-POSEDNESS, ILL-POSEDNESS AND NORM-INFLATION FOR A
HIGHER ORDER WATER WAVE MODEL ON A PERIODIC DOMAIN**

ABSTRACT. In this work we are interested in the well-posedness issues for the initial value problem (IVP) associated with the fifth order KdV-BBM type model

$$\eta_t + \eta_x - \gamma_1 \eta_{xxt} + \gamma_2 \eta_{xxx} + \delta_1 \eta_{xxxxt} + \delta_2 \eta_{xxxxx} + \frac{3}{2} \eta \eta_x + \gamma (\eta^2)_{xxx} - \frac{7}{48} (\eta_x^2)_x - \frac{1}{8} (\eta^3)_x = 0,$$

posed on a periodic domain \mathbb{T} . We derive some multilinear estimates and use those in the contraction mapping argument to prove the local well-posedness for initial data in the periodic Sobolev space $H^s(\mathbb{T})$, $s \geq 1$. With restriction on the parameters appeared in the model, we use the conserved quantity to obtain global well-posedness for given data with Sobolev regularity $s \geq 2$. Also, we use splitting argument to obtain the global well-posedness result when $1 \leq s < 2$. We also show that the well-posedness result obtained in this work is sharp by showing that the flow-map that takes initial data to the solution cannot to be continuous for given data in $H^s(\mathbb{T})$, $s < 1$. Finally, for $s < 1$, we prove a norm-inflation result by showing that the solution corresponding to a smooth initial data may have arbitrarily large $H^s(\mathbb{T})$ norm for arbitrarily short time.

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