EXISTENCE OF MULTIPLE SOLUTIONS FOR THE VAN DER WAALS-CAHN-HILLIARD EQUATION

Abstract. In this talk we discuss the existence of multiple solutions for the following nonlinear problem: for a fixed $V \in \mathbb{R}^+ =]0, +\infty[$ and $\varepsilon > 0$ small, find $u \in H^1_0(\Omega)$, and $\lambda \in \mathbb{R}$ such that

$$-\varepsilon^2 \Delta u + W'(u) = \lambda,$$

and

$$\int_\Omega u(x)\, \mathrm{d}x = V,$$

where Ω is an open bounded set in $\mathbb{R}^N$ with Lipshitz boundary and $W : \mathbb{R} \to \mathbb{R}$ is a Double Well potential of class C^2 which satisfies the following assumptions:

(a) $W(0) = W'(0) = 0$, $W''(0) > 0$;

(b) there exists $s_0 \in]0, +\infty[$ such that

$$W(s_0) = \min\big\{W(s) : s \in \mathbb{R}\big\} < 0;$$

(c) suitable growth conditions.

The simplest example of this type of potentials is given by the non-symmetric Allen-Cahn potential:

$$W(s) = s^2(s - s_1)(s - s_2),$$

where $0 < s_1 < s_0 < s_2$. A lower bound on the number of solutions is estimated in terms of topological and homological invariants of the underlying domain Ω, coming from suitable abstract theorems due to the first author and colaborators. In theoretical biology equations of this type model pattern formation related to solutions which are not absolute minima of the energy. From a purely mathematical viewpoint, the above equation is also interesting due to its relation with the theory of constant mean curvature hypersurfaces. This is a joint work with V. Benci and P. Piccione.

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