ON THE EXISTENCE AND ASYMPTOTIC PROPERTIES FOR A DISSIPATIVE SEMILINEAR SECOND ORDER EVOLUTION EQUATIONS WITH FRACTIONAL LAPLACIAN OPERATORS

Abstract. We study the existence and uniqueness of solutions to the following second order semilinear evolution equation with Laplace operators depending on fractional powers. We also study the decay rates of the solution and associated energy in the sense of L^2 norm. For the associated linear equation, using an asymptotic expansion of the solution to the problem in Fourier space we show optimality of the decay rates obtained for certain powers of the Laplacian operators . Moreover, we study the case of super damping to the linear problem ($\beta = 0$) using an improvement of the standard case and we also prove optimality of the decay rates for L^2 norms. Our results generalize some previous works that deal with particular cases of the fractional exponents of the Laplacian operator.

$$\begin{cases} u_{tt} + (-\Delta)^{\delta} u_{tt} + (-\Delta)^{\alpha} u + (-\Delta)^{\theta} u_t = \beta(-\Delta)^{\gamma}(f(u)), \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x). \end{cases} \tag{1}$$

In the Cauchy problem (1) β is a real constant, $u = u(t,x)$ with $(t,x) \in (0,\infty) \times \mathbb{R}^n$ and the exponents of the Laplacian operators are constants satisfying $0 \leq \delta \leq \alpha$, $0 \leq \theta \leq \frac{\alpha+\delta}{2}$ and $\max\{ 0, \frac{\alpha}{2} - \frac{n}{4}\} \leq \gamma \leq \frac{\alpha+\delta}{2}$. The nonlinearity f behaves as $f(s) = s^p$ with $p > 1$ an integer. To show that the decay rates are optimal we use asymptotic expansion of the solution and its derivatives in Fourier space.

Authors.

(1) Ruy Coimbra Charão. *Federal University of Santa Catarina*
E-mail: *ruy.charao@ufsc.br*

(2) Juan Carlos Torres Espinoza. *Federal University of Santa Catarina*
E-mail: *juantorres1507@gmail.com*

(3) Ryo Ikehata . *Hiroshima University*
E-mail: *ikehatar@hiroshima-u.ac.jp*

References.

(1) Horbach, J. L.; Ikehata, R.; Charão, R. C.; *Optimal Decay Rates and Asymptotic Profi le for the Plate Equation with Structural Damping*, J. Math. Analysis Appl. 440, 2016, 529-560.

(2) Ikehata, R.; Iyota, S.; *Asymptotic profile of solutions for some wave equations with very strong structural damping*, Math. Meth. Appl. Sci. **41** (2018), 5074–5090.

(3) Luz, C. R. da; Ikehata, R. ; Charão, R. C.; *Asymptotic behavior for abstract evolution differential equations of second order*, Journal Differential Equations 259, 2015, 5017-5039.

(4) Wang, S.; Xu, H.; *On the asymptotic behavior of solution for the generalized IBq equation with hydrodynamical damped term*, J. Differential Equations 252, 2012, 4243-4258.