SHARP ILL-POSEDNESS AND WELLPOSEDNESS RESULTS FOT THE GENERALIZED KDV-B EQUATION ON THE REAL LINE

Abstract. In this paper we study the well-posedness of the generalized Korteweg-De Vries- Burgers equation

$$\partial_t u + \partial_x^3 u + L_p u + u\partial_x u = 0, \quad x \in \mathbb{R},\ t \geq 0, \qquad \text{(g-KdV-B)}$$

where $u = u(t,x)$ is a real-valued function and $\mathcal{F}_x\{L_p u\}(t,\xi) = |\xi|^p \mathcal{F}_x u(t,\xi)$, for $p \in \mathbb{R}^+$. When $p = 2$ we have the well-known KdV - Burgers equation. This equation arises in some different physical contexts as a model equation involving the effects of dispersion, dissipation and nonlinearity. When $p = 1/2$ the related equation models the evolution of the free surface for shallow water waves damped by viscosity.

The well-posedness for the equation (g-KdV-B) has been studied for many authors. In 2001, using the Bourgain spaces, related only to the KdV equation (see e.g. [1] and [2]), and the bilinear estimate due to Kenig, Ponce and Vega (see [3]), Molinet and Ribaud obtained the global well-posedness (g.w.p) in $H^s(\mathbb{R})$, for $s > -3/4$ and $p > 0$. In the particular case of $p = 2$ (KdV-Burgers equation), they proved g.w.p. in $H^s(\mathbb{R})$, for $s > -3/4 - 1/24$ (see [4])

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