

## THE STOCHASTIC BIDOMAIN PROBLEM

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ABSTRACT. The bidomain problem models the electrical properties of heart muscle and describes the intra- and extracellular electric potentials  $u_1$  and  $u_2$  and the transmembrane potential  $u = u_1 - u_2$ . We consider this model with FitzHugh-Nagumo transport and a stochastic force, i.e., the set of equations

$$\begin{aligned}\partial_t u - \operatorname{div} (a_1 \nabla u_1) &= -u^3 + (a + 1)u^2 - au - w + \xi, \\ \partial_t u + \operatorname{div} (a_2 \nabla u_2) &= -u^3 + (a + 1)u^2 - au - w + \xi, \\ \partial_t w &= -bw + cu, \\ u &= u_1 - u_2,\end{aligned}$$

where  $\xi$  is a given noise term,  $0 < a < 1$ ,  $b, c > 0$ ,  $a_1, a_2 \in W^{1,\infty}$  and  $w$  is the so called recovery variable. The linear part of this system is described by an operator, which admits a bounded  $H^\infty$ -calculus. In the recent work (1) by Hieber and Pr this was used to prove the global existence of solutions for rather rough initial data by deducing energy estimates and applying the theory of critical spaces from (2). We show how to combine their argument with a result on stochastic maximal regularity (see (3)) to obtain a global solution to the bidomain problem with additive noise.

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**Authors.**

- (1) MARTIN SAAL. *TU Darmstadt*.  
**E-mail:** [msaal@mathematik.tu-darmstadt.de](mailto:msaal@mathematik.tu-darmstadt.de)
- (2) MATTHIAS HIEBER. *TU Darmstadt*.  
**E-mail:** [hieber@mathematik.tu-darmstadt.de](mailto:hieber@mathematik.tu-darmstadt.de)
- (3) AMRU HUSSEIN. *TU Darmstadt*.  
**E-mail:** [hussain@mathematik.tu-darmstadt.de](mailto:hussain@mathematik.tu-darmstadt.de)

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