PRIMAL HYBRID FINITE ELEMENT METHODS APPLIED TO THE HELMHOLTZ EQUATION

Abstract. Phenomena of acoustic, electromagnetic or elastic waves are present in many areas of science, engineering and industry. Physicists and engineers often need to simulate wave propagation processes. These oscillations are related in such a way that energy is propagated in the medium.

Assuming a linear constitutive law and considering the propagation of harmonic waves over time, we obtain the Helmholtz equation whose solutions depend on a parameter κ, called wave number (1), which characterizes the frequency oscillations of the harmonic solutions. In our case we study the Helmholtz equation with Robin conditions (1)-(2),

$$-\Delta p - \kappa^2 p = f, \quad \text{on } \Omega \tag{1}$$

$$-\nabla p \cdot \mathbf{n} + \mathrm{i}\kappa p = g, \quad \text{in } \partial\Omega \tag{2}$$

It is a major challenge the development of computationally efficient robust numerical methods for harmonic waves over time, governed by the Helmholtz equation with high wave numbers. The quality of the numerical solution depends on the number of waves κ. For high wave numbers (high frequencies) the associated differential operator becomes indefinite thus compromising the stability of the approximations by Galerkin's classical finite element or finite difference methods. As analyzed by Ihlenburg and Babuska (2), the finite element method with linear approximations present adequate asymptotic behavior, with optimal convergence rate, only for extremely refined meshes, which obey the condition $\kappa^2 h \leq 1$, which makes this approximation unfeasible for Real problems with high numbers of waves κ.

Estimates of the asymptotic error, respecting the restriction $\kappa^2 h \leq 1$, have been obtained for classical approximations via the Galerkin method. Fundamental results were also obtained for $\kappa h \leq 1$, referred to as pre-asymptotic behavior. In this work, we propose new primal hybrid finite element methods for the Helmholtz equation $((3), (4), (5))$.

Key words: Helmholtz equation, Finite elements, Galerkin methods Discontinuous, Hybrid Methods and Error Estimation.

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