APPLICATION TO FINANCE: PARTIAL DIFFERENTIAL EQUATION OF BLACK AND SCHOLES

ABSTRACT. In the Black and Scholes economy,the value V_t of a financial derivative is expressed as a linear combination of an underlying asset (risky) S_t and a risk-free asset B_t. In this paper, under the assumptions of the Black and Scholes economy and that the economic agent continuously learns and negotiates; and by using stochastic differential equations we will show that the value of a financial derivative V_t satisfies the following partial differential equation (PDE):

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S_t^2} + (r-q)S_t \frac{\partial V}{\partial S_t} - rV_t = 0. \tag{1}$$

where:

S_t satisfies the following stochastic equation

$$dS_t = \mu S_t dt + \sigma dW_t \tag{2}$$

r : risk free rate.
σ: volatility of the underlying asset (risk).
q : dividend payment of the underlying asset during the life of the derivative.
μ: average rate of return of the underlying asset.
W_t: Brownian motion.

To demostrate equation (1), called Black & Scholes PDE, the synthetic replication method will be used. As a particular case and application, the solution will be shown when it comes to financial derivatives: Call option and Put option, whose solutions are given by:

$$C_t = N(d_1)S_t - Ke^{-r(T-t)}N(d_2)$$
$$P_t = -N(-d_1)S_t - Ke^{-r(T-t)}N(-d_2),$$

where:

$$d_1 = \frac{ln(S_t/K) + (T-t)(r-q+\sigma^2/2)}{\sigma\sqrt{T-t}} \text{ and } d_1 - d_2 = \sigma\sqrt{T-t}$$

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