

**$\sigma$ -EVOLUTION MODELS WITH LOW REGULAR TIME DEPENDENT  
EFFECTIVE STRUCTURAL DAMPING**

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ABSTRACT. In this talk, we discuss decay estimates for solutions to the  $\sigma$ -evolution model:

$$u_{tt}(t, x) + A^\sigma u(t, x) + b(t)A^\theta u_t(t, x) = 0, \quad (t, x) \in (0, \infty) \times \mathbf{R}^n \quad (1)$$

with suitable initial data

$$u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x) \quad x \in \mathbf{R}^n, \quad (2)$$

in which  $A := -\Delta$ ,  $\sigma > 0$ ,  $\theta \in [0, \sigma]$  and with low regular  $b$ , that is, we don't assume any control of  $\frac{d}{dt}b$ , a relevant difference in comparison with other works, such [1].

In our previous work [2], we developed a new energy method to treat the problem (1)-(2) in the non-effective case. Originally, the idea was to separate  $[t_0, \infty) \times \mathbf{R}^n$  in several zones, say  $Z$ , and for each of them find a suitable  $K(t, \xi)$  such that

$$\int_S^T K(s, \xi) E(s) ds \leq CE(S), \quad (3)$$

for all  $(S, \xi)$  and  $(T, \xi)$  in  $Z$ . Here,  $E$  represents the density energy  $E(t) = E(t, \xi)$  associated with (1) in Fourier space. The monotonicity of  $E$  and inequality (3), ensure decay rates for the problem (1)-(2). On the other hand, in the effective case a straightforward application of this method is possible, but is not enough to ensure the sharp decay rates already known for the case in which  $b$  is more regular [1]. To overcome this issue we try to replace the standard energy  $E$  in inequality (3) by a suitable function  $F(t, \xi)$ .

Roughly speaking, it seems that we will choose  $F$  as Lyapunov's functional for  $E$ . However, that is not exactly the point. Firstly, our  $F(t, \xi)$  is not equivalent to  $E(t, \xi)$ . Secondly, we only have some control on  $\frac{d}{dt}F(t)$ , but no monotonicity for  $F$  is available. With  $F$  we obtain decay estimates for  $u$ , and using it further steps ensure decay estimates for  $u_t$ . We shall see that this construction is far from easy and several additional results, when compared to [2], are required to achieve the desired estimates in the effective case.

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