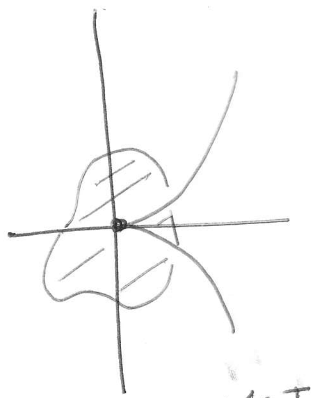
1

$\gamma(t) = (t^2, t^3)$

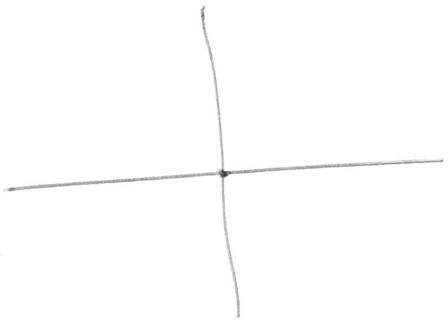


$f : \Omega \to \mathbb{R}$ sub

$\gamma(I) = f^{-1}(\{0\})$

$\gamma(I) = X$

$(f \circ \gamma)$



$\Rightarrow Df(0) = 0$

$\frac{d}{dt}(f\circ\gamma)(t)$

$= \cancel{\frac{\partial f}{\partial x}(\gamma(t))\, 2t} + \frac{\partial f}{\partial y}(\gamma(t))\, 3t^2$

$\frac{d^2}{dt^2}(f\circ\gamma)(t)$

$= \cancel{\dots} + \cancel{2\frac{\partial f}{\partial x}(\gamma(t))}$

$+ O(t^2) + 6t\, \frac{\partial^2 f}{\partial y}(\gamma(t))$

$\Big|_0 = 2\frac{\partial f}{\partial x}\ (\neq 0)$

$\frac{d^3}{dt^3}(f\circ\gamma)(t)\Big|_0 = 6\frac{\partial f}{\partial y}(0) = 0$

2/

$\cancel{\forall i}$ $\quad \gamma : [\cancel{0}\, a, b] \to \mathbb{R}^m$

$\#$ $\quad a = t_0 < t_1 < \dots < t_k = b$

$$\tilde{L}[\gamma] := \sum_{i=1}^{k-1} \| \gamma(t_{i+1}) - \gamma(t_i) \|$$

$$\forall i \quad \| \gamma(t_{i+1}) - \gamma(t_i) \| \cancel{=} \leq \int_{t_i}^{t_{i+1}} \| \dot{\gamma}(t) \| \, dt$$

<u>CONVEXIDADE</u>

$$\| \gamma(t_{i+1}) - \gamma(t_i) \| = \left\| \int_{t_i}^{t_{i+1}} \dot{\gamma}(t) \, dt \right\|$$

$$\leq \int_{t_i}^{t_{i+1}} \| \dot{\gamma}(t) \| \, dt \quad \checkmark$$

$$\sum_{i=1}^{k-1} \| \gamma(t_{i+1}) - \gamma(t_i) \| \leq \sum_{i=1}^{k-1} \int_{t_i}^{t_{i+1}} \| \dot{\gamma}(t) \| \, dt$$

$$= \int_a^b \| \dot{\gamma}(t) \| \, dt$$

$$L[\gamma] \leq \int_a^b \| \dot{\gamma}(t) \| \, dt$$

$$\int_a^b \|\dot\gamma(t)\|\,dt \leq L[\gamma]$$

$\nexists$ $\forall \varepsilon > 0$ $\exists\, t_0, t_1 \dots t_k$ TQ.

$$\sum_{i=1}^{k-1} \|\gamma(t_{i+1}) - \cancel{t_i}\, \gamma(t_i)\| \geq \int_a^b \|\dot\gamma(t)\|\,dt - \varepsilon$$

$\ddot\gamma$ CONT. $\exists B$ TQ. $\|\ddot\gamma(t)\| \leq B \quad \forall t$

$$\underbrace{\gamma(t_{i+1}) - \gamma(t_i)}_{z} = \underbrace{(t_{i+1} - t_i)\dot\gamma(t_i)}_{y} + \underbrace{\int_{t_i}^{t_{i+1}} (t_{i+1} - s)\ddot\gamma(s)\,ds}_{x}$$

$$\Rightarrow \Big|\, \underbrace{\|\gamma(t_{i+1}) - \gamma(t_i)\|}_{z} - \# \underbrace{|t_{i+1} - t_i|\, \|\dot\gamma(t_i)\|}_{y} \Big|$$

$$\leq \underbrace{\left\| \int_{t_i}^{t_{i+1}} (t_{i+1} - s)\ddot\gamma(s) \mp ds \right\|}_{x}$$

DES. Δ.

$$\leq \frac{1}{2} B\, |t_{i+1} - t_i|^2$$

$$\Rightarrow \left| \sum_{i=0}^{k-1} \| \gamma(t_{i+1}) - \gamma(t_i) \| \right.$$ $\longrightarrow$ SOMA DE R

$$\left. - \underbrace{\sum_{i=0}^{k-1} (t_{i+1} - t_i) \| \dot{\gamma}(t_i) \|} \right| \qquad \downarrow \int_a^b \| \dot{\gamma}(t) \| dt$$

$$\leq \frac{1}{2} B \sum_{i=0}^{k-1} | t_{i+1} - t_i |^2$$

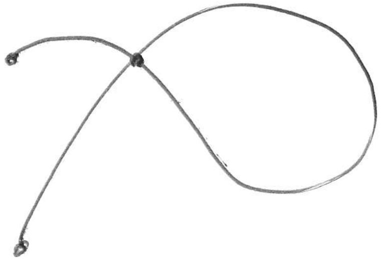
$$t_i := a + \frac{i(b-a)}{k} \qquad k >> 1$$

$$\leq \frac{B}{2k} \longrightarrow 0 \qquad k \to \infty$$

k SUFF. GRANDE

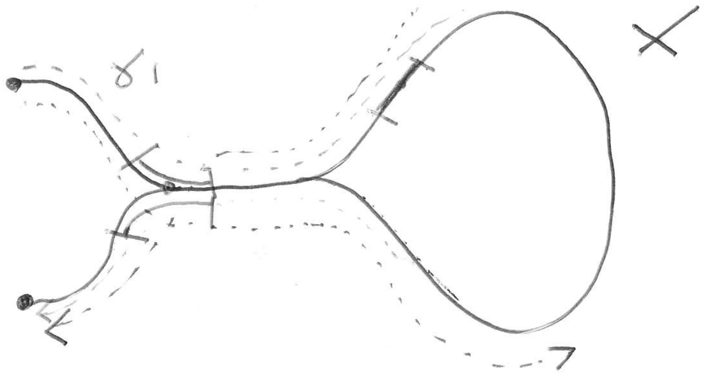
$$\sum_{i=0}^{k-1} \| \gamma(t_{i+1}) - \gamma(t_i) \| \geq \int_a^b \| \dot{\gamma}(t) \| dt - \varepsilon$$

$$\Rightarrow \mathcal{L}[\gamma] \geq \int_a^b \| \dot{\gamma}(t) \| dt \cancel{- \varepsilon}$$



$\gamma_1 : [a, b] \longrightarrow \mathbb{R}^2$

INJ. HOMEO.

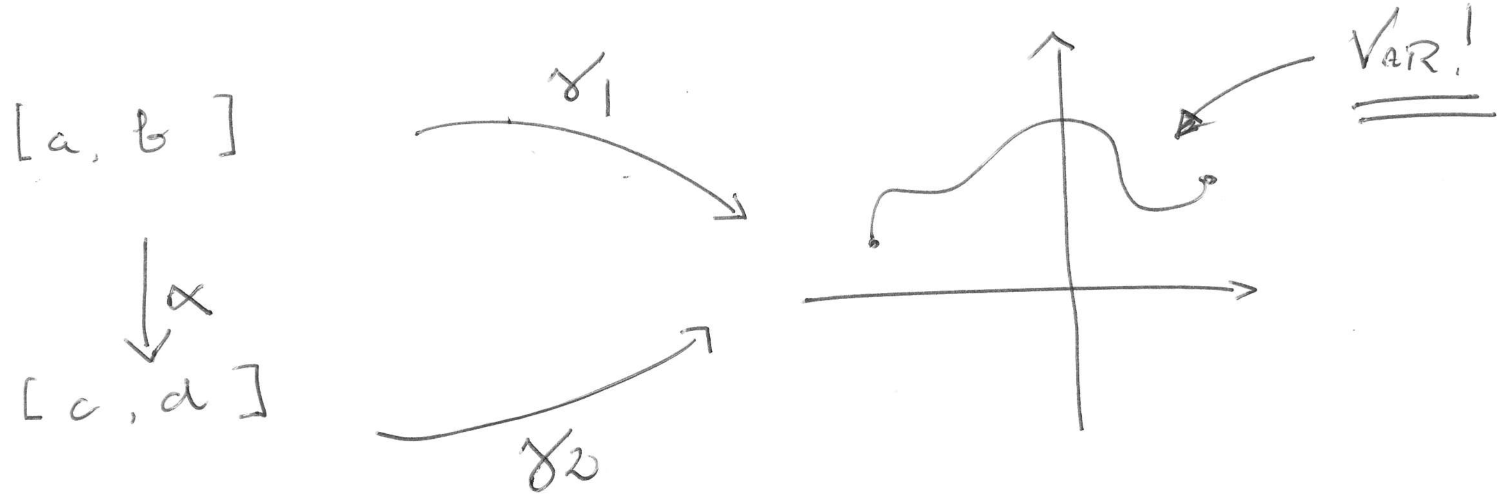


$\gamma_1 : [a, b] \to \mathbb{R}^2$

$\gamma_2 : [c, d] \to \mathbb{R}^2$

$\gamma_2([c, d]) = \gamma_1([a, b])$

7/



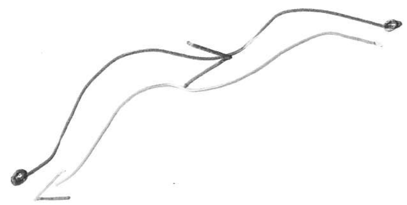
$$\gamma_2 \circ \alpha = \gamma_1$$

$$\gamma_1([a,b]) = \gamma_2([c,d])$$

$\gamma_1 :]a, b[\to \mathbb{R}^n$ $\qquad$ $\gamma_2 :]c, d[\to \mathbb{R}^n$

$\| \dot{\gamma}_1(t) \| = \| \dot{\gamma}_2(t) \| = 1$

$\alpha :]a, b[\to]c, d[$ TQ.



$$\gamma_1 = \gamma_2 \circ \alpha$$

$\dot{\gamma}_1(t) = \dot{\gamma}_2(\alpha(t)) \cdot \dot{\alpha}(t)$

$\Rightarrow 1 = 1 \cdot |\dot{\alpha}(t)|$

$\dot{\alpha}(t) \in \{\pm 1\}$ $\qquad$ $\dot{\alpha}(t) = 1$ $\qquad$ $\dot{\alpha}(t) = -1$

- $\alpha(t) = t + m$ $\qquad$ $\gamma_1(t) = \gamma_2(t + m)$

$\alpha(t) = -t + m$ $\qquad$ $\gamma_1(t) = \gamma_2(-t + m)$

$\gamma :]a, b[\longrightarrow \mathbb{R}^m$

REG! $\Rightarrow \quad \|\dot{\gamma}(t)\| > 0$

$\beta(t) = \int_c^t \|\dot{\gamma}(t)\| \, dt \qquad c \in]a, b[$

$\dot{\beta} = \underline{\|\dot{\gamma}\|} \qquad \|\cdot\|$ SUAVE SOBRE $\mathbb{R}^m \setminus \{0\}$

SUAVE

$\dot{\beta} > 0$ CRESCENTE $\Rightarrow$ inv.

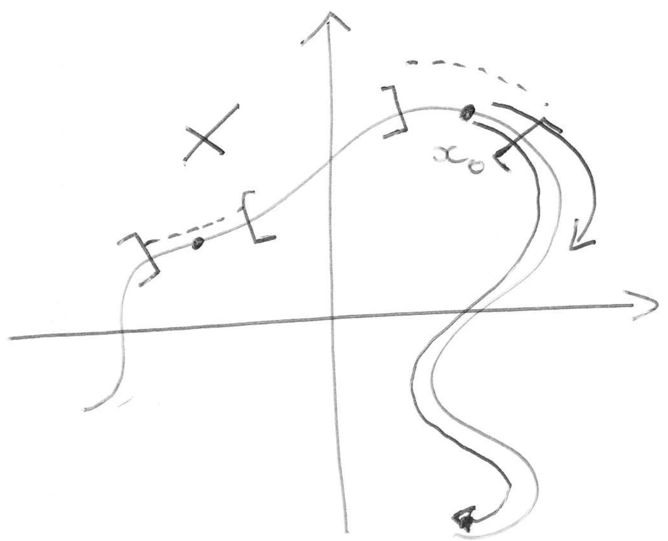
$]c, d[= \mathrm{Im}(\beta) \qquad \alpha := \beta^{-1}$

$\nu(t) := (\gamma \circ \alpha)(t)$

$$\dot{\omega}(t) = \dot{\gamma}(\alpha(t)) \cdot \dot{\alpha}(t)$$

$$= \dot{\gamma}(\alpha(t)) \cdot \frac{1}{\dot{\beta}(\alpha(t))}$$

$$\Rightarrow \|\dot{\omega}(t)\| = \|\dot{\gamma}(\alpha(t))\| / \|\dot{\gamma}(\alpha(t))\| = 1$$



~~COMPACTA~~
CONNEXA
$\Rightarrow \exists$ PARAM.
GLOBAL